Calibration of Preston Tubes in Supersonic Flow

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Nomenclature

τ_0 = wall shearing stress
d = outside diameter of Preston tube
ρ^* = density of fluid evaluated at reference temperature
ν^* = kinematic viscosity of fluid evaluated at reference temperature
U = velocity ahead of the Preston tube as obtained from the Rayleigh Pitot formula
i_1 = enthalpy in the freestream
r = recovery factor
γ = ratio of the specific heats
M = freestream Mach number

Introduction

HYPOTHESIS of a reference temperature, or enthalpy, has proved of great value in the development of rapid and accurate methods for calculation of turbulent skin friction in supersonic boundary layers.[1] The purpose of this note is to show that this same hypothesis can be used to obtain a calibration formula for Preston tubes that may be applied to both compressible and incompressible flow.

Discussion

It will be recalled that the Preston tube is a small total head tube aligned in the flow direction inside a boundary layer. Preston[2] has shown that it is possible to obtain local values of the shear stress from the reading of the total head tube. Since then, when used in that fashion, the instrument has often been referred to as a Preston tube. The method is based on the universal law of the wall for turbulent boundary layers, and its validity now has been well demonstrated.[3,4]

A calibration of Preston tubes in supersonic flow was first obtained by Fenter and Stalmach.[5] Figure 1 shows how the data from Ref. 5 can be used to obtain a calibration formula by means of the reference temperature hypothesis. It also shows a relationship between two nondimensional groups of parameters $\tau_0 d^2/4\rho^*\nu^{*2}$ and Ud/ν^*.

The reference temperature is calculated (for an adiabatic wall) by means of the formula given in Ref. 6 for the reference enthalpy i^*, $i^* = i_1[1 + 0.35r\ (\gamma - 1)M^2]$. The points shown in Fig. 1 have been obtained directly from Fig. 15-A of Ref. 5. The data shown were obtained with a number of different Preston-tube diameters in a Mach number range from 1.74 to 3.68. The line drawn through the points in Fig. 1 may be represented by

$$(\rho^* d^2 \tau_0)/\mu^{*2} = 0.0529[(\rho^* d^2 \Delta p)/\mu^{*2}]^{0.873} \tag{1}$$

where Δp is related to U by $\Delta p = \frac{1}{2}\rho^* U^2$, and $\mu^* = \nu^*\rho^*$. In incompressible flow Δp would be the difference between the total pressure reading of the Preston tube and the local static pressure. The calibration from Fig. 1 is presented in Eq. (1) so that it may be compared directly to the calibrations of Preston tubes in incompressible flows given in Eqs. (2–4):

$$(\rho d^2 \tau_0)/\mu^2 = 0.0478[(\rho d^2 \Delta p)/\mu^2]^{0.875} \tag{2}$$

$$(\rho d^2 \tau_0)/\mu^2 = 0.0543[(\rho d^2 \Delta p)/\mu^2]^{0.875} \tag{3}$$

$$(\rho d^2 \tau_0)/\mu^2 = 0.0511[(\rho d^2 \Delta p)/\mu^2]^{0.877} \tag{4}$$

These equations were obtained by Preston,[1] Relf et al.,[7] and Smith and Walker,[8] respectively. It can be seen that the calibration formula obtained from the supersonic data by means of the reference temperature hypothesis is in very good

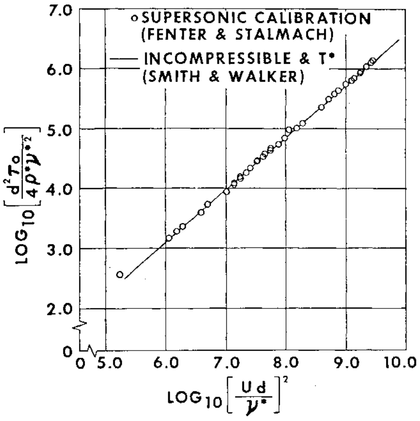

Fig. 1 Supersonic calibration of Preston tubes.

agreement with calibration formulas obtained in incompressible flow. The result indicates that the reference temperature concept is not only useful for the calculation of supersonic skin friction, but can also be profitably applied to the analysis of other aspects of compressible boundary-layer flow.

Received April 13, 1965.
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Blunt-Body Integral Method for Air in Thermodynamic Equilibrium

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THE direct (Dorodnitsyn-Belotserkovskii) integral method has been adapted for application to an adiabatic flow process involving an arbitrary smooth axisymmetric or two-dimensional blunt-body in supersonic flow. The gas under consideration is argon-free air in thermodynamic equilibrium.

Received January 11, 1965; revision received March 29, 1965.
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In order to provide the necessary thermodynamic relationships, a functional approximation of the data of Refs. 1 and 2, in terms of polynomial surface fits, was employed. The relations used in the present analysis take the form

$$\rho = \rho(h, p) \qquad S = S(h, p) \qquad p = p(h, S) \qquad a = a(h, S) \tag{1}$$

where h = enthalpy, S = nondimensional entropy, and $\Delta S/\mathcal{R}$ and a = speed of sound. In addition the energy equation is given by

$$h + \tfrac{1}{2}(v_s^2 + v_n^2) = h_t \tag{2}$$

where v_s and v_n are tangential and normal velocity components, respectively, in a body-oriented, curvilinear coordinate system. An empirical subroutine describing the thermodynamic properties of air from 90° to 15,000°K (Ref. 3) was used as an additional source of data.

Development of Governing Equations

Initial efforts to represent the governing partial differential equations in "divergence" form involved direct use of the continuity equation. This approach avoids the particle-isentropic assumption implicit in the "modified" continuity equation employed in the original Belotserkovskii analysis, i.e.,

$$(\partial/\partial s)[r^j \tau v_s] + (\partial/\partial n)[r^j \tau (1 + n/R) v_n] = 0 \tag{3}$$

However, the resulting description of the shock and body parameters becomes progressively less accurate with increasing Mach number in the supersonic regime, e.g., the shock lies too close to the body.[4] Since the original formulation yields valid results in the regime where real gas effects are not dominant, it was then required that the present analysis reduce identically to this version under perfect gas assumptions. Use of a more general form of the "modified" continuity equation accomplished this purpose, i.e.,

$$(\partial/\partial s)[r^j \rho e^S v_s] + (\partial/\partial n)[r^j \rho e^S (1 + n/R) v_n] = 0 \tag{4}$$

Comparing Eqs. (3) and (4), it is evident that $\tau = \rho e^S$. Since

$$\tau = \rho/\rho_{t\infty}\, \phi^{1/(\gamma - 1)} \qquad \phi = [p/p_{t\infty}]/[\rho/\rho_{t\infty}]^\gamma$$
$$[S]_{\text{perfect}} = 1/(\gamma - 1) \ln_e \phi$$

the formulations become identical for a perfect gas.

The governing differential equations for the shock-layer thickness measured normal to the body surface δ, shock angle χ, and surface velocity v_{s0} are given by

$$F_1 + F_2 (d\delta/ds) = F_3 (d\chi/ds) \tag{5}$$

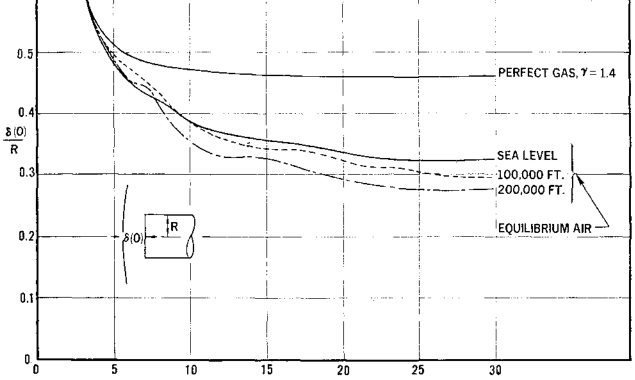

Fig. 1 Shock detachment distance (flat-faced cylinder).

$$F_4 + F_5 \frac{d\delta}{ds} + F_6 \frac{d\chi}{ds} = \left(1 - \frac{v_{s0}^2}{a_0^2}\right) \frac{dv_{s0}}{ds} \tag{6}$$

where

$$\frac{d\delta}{ds} = \left(1 + \frac{\delta}{R}\right) \operatorname{ctn}(\chi + \theta) \qquad r_\delta{}^j = r_0{}^j + j\delta \cos\theta$$

$$F_1 = \delta\left\{\rho_\delta \left[\frac{r_\delta{}^j}{R} v_{n\delta}{}^2 - j v_{s\delta} v_{n\delta} \sin\theta \left(1 + \frac{\delta}{R}\right)\right] + \frac{r_\delta{}^j}{R} p_\delta + \frac{r_0{}^{2j}}{R} (\rho_0 v_{s0}{}^2 + p_0) + j \cos\theta \left[p_0 + p_\delta \left(1 + \frac{\delta}{R}\right)\right]\right\} + 2\left[r_0{}^j p_0 - r_\delta{}^j (\rho_\delta v_{n\delta}{}^2 + p_\delta)\left(1 + \frac{\delta}{R}\right)\right]$$

$$F_2 = r_0{}^j \rho_\delta v_{s\delta} v_{n\delta} \qquad F_3 = r_\delta{}^j \delta (d/d\chi)(\rho_\delta v_{s\delta} v_{n\delta})$$

$$F_4 = -\left(\frac{2}{\delta} + \frac{1}{R}\right) \frac{r_\delta v_{n\delta}}{r_0 \rho_0} \xi - j \frac{\sin\theta}{r_0} \left[\left(1 + \frac{\delta}{R}\right) \frac{v_{s\delta}}{\rho_0} \xi + v_{s0}\right]$$

$$F_5 = \frac{1}{\delta}\left(\frac{v_{s\delta}}{\rho_0} \xi - v_{s0}\right) \qquad \xi = \rho_\delta \exp(S_\delta - S_0)$$

$$F_6 = -\frac{r_\delta \xi}{r_0 \rho_0} \left[\left(\frac{1}{\rho_\delta} \frac{d\rho_\delta}{d\chi} + \frac{dS_\delta}{d\chi}\right) v_{s\delta} + \frac{dv_{s\delta}}{d\chi}\right]$$

The subscripts 0 and δ denote body and shock values, respectively. For the axisymmetric case ($j = 1$), initial values of the derivatives $d\chi/ds$ and dv_{s0}/ds were obtained by means of a limiting procedure for $s \to 0$:

$$\left[\frac{d\chi}{ds}\right]_0 = \left[\frac{p_0 - p_\delta - \rho_\delta v_{n\delta}{}^2}{\delta \rho_\delta (V_\infty + v_{n\delta}) v_{n\delta}}\right]_{s=0}$$

$$\left[\frac{dv_{s0}}{ds}\right]_0 = \left[\left(1 + \frac{\delta}{R}\right) \frac{p_\delta - p_0}{\delta \rho_0 v_{n\delta}}\right]_{s=0}$$

For the two-dimensional case ($j = 0$), no indeterminateness (0/0) arises. An explicit representation of the flow properties in terms of the dependent variables χ and v_{s0} is not possible as in the perfect gas case. Therefore, a table of post-shock flow properties as a function of χ was generated through an iteration involving the shock conservation equations and the thermodynamic relationships. On the body surface, use of Eqs. (1) and (2) and knowledge of S_0 (from the aforementioned table for $\chi = 0$) enables one to obtain the surface properties as functions of v_{s0}. The derivatives $dS_\delta/d\chi$ and $d\theta_\delta/d\chi$ (θ_δ = flow deflection angle) are also generated numerically thus enabling the subsequent evaluation of $dv_{s\delta}/d\chi$, $dv_{n\delta}/d\chi$, and $d\rho_\delta/d\chi$ using oblique shock conservation relations.

Numerical Results

Flat-Faced cylinder

Although the integration of the governing system of equations for general body shapes comprises a two-point boundary value problem, the particular case of a flat-faced cylinder reduces to an initial value problem. This results from the lack of coupling between δ and the geometrical parameters as the integration progresses, since $s \equiv r$, $\theta = \pi/2$, $R^{-1} = 0$. The location of the body sonic point, $s_0{}^* = r_0{}^*$ (defined by $v_{s0} = a_0{}^*$), therefore becomes purely a function of $\delta(0)$ such that the ratio $\delta(0)/r^*$ remains constant for given freestream conditions. The corner radius may therefore be scaled to some convenient value, say unity.

The variation of shock detachment distance with Mach number and altitude, presented in Fig. 1, exhibits a significant departure from perfect gas results. The irregular behavior of the equilibrium curves is presumed to be related to the nonmonotonic variation with Mach number and altitude of such post-shock quantities as ρ_δ and V_δ.

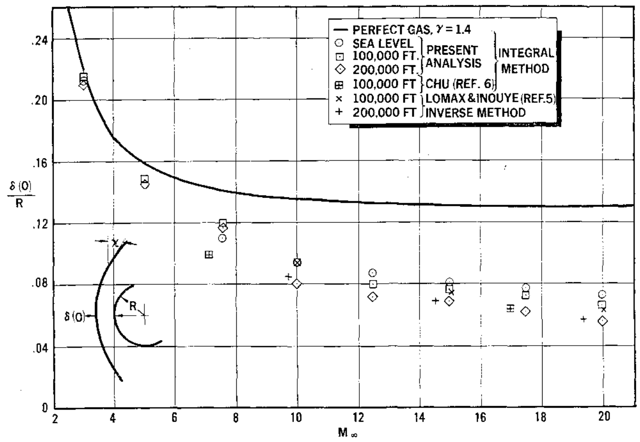

Fig. 2 Shock detachment distance (sphere).

Sphere

As in the perfect gas case, the sphere calculation involves adjustment of the shock detachment distance until the singular behavior of the expression for dv_{s0}/ds is restricted to a sufficiently small neighborhood of the body sonic point. An additional factor, introduced in the equilibrium case, which influences convergence is the smoothness of the thermodynamic data, i.e., slope discontinuities in curve or surface representations of the data are not allowable in the critical sonic region. Since a given Mollier surface is composed of a number of segments "patched" together, discontinuity problems may arise at the boundaries of the segments. Nonconvergent cases were, in fact, encountered for certain free-stream conditions employing both thermodynamic subroutines. The Arnold Engineering Development Center routine was used predominantly for the sphere calculations, owing to its smoother surface transitions.

Figure 2 presents the variation of shock detachment distance with Mach number and altitude for the sphere case. Equilibrium air results are plotted as discrete points, since the nonmonotonic behavior would require a large number of computed cases to establish continuous curves. Inverse method detachment distances[5] are in substantial agreement with present results. Reference 6 also employs the direct integral method; however, the governing equations and thermodynamic procedures differ from those of the present analysis, e.g., the differential equation for the surface velocity [Eq. (6)] is obtained by integration of the unmodified continuity equation.

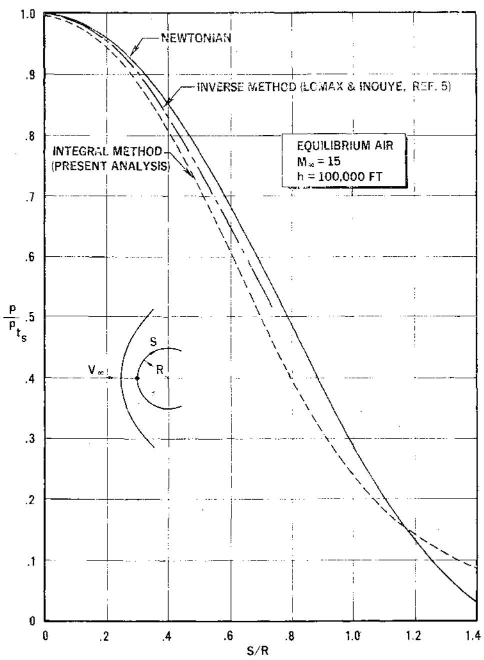

Fig. 3 Surface pressure distribution (sphere).

A comparison of representative sphere pressure distributions is shown in Fig. 3. Perfect gas results for the direct and inverse method were found to coincide closely with respective equilibrium curves and are therefore not indicated.

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Modified Blast-Analogy for the Blunt Flat Plate with Sweep and Attack

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The analogy to an explosion has been applied to simple blunt shapes in hypersonic flow with a moderate degree of success. For an unswept plate at zero angle of attack the

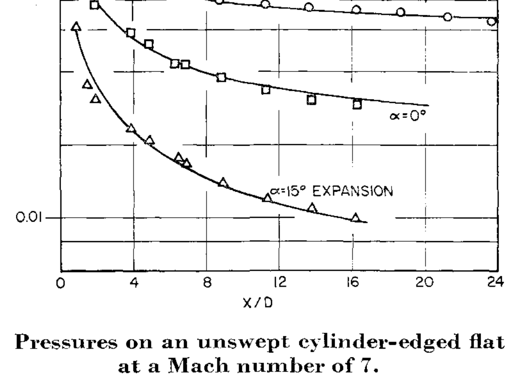

Fig. 1 Pressures on an unswept cylinder-edged flat plate at a Mach number of 7.

Received April 2, 1965.

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